Channel Coding

The purpose of this project is write a program, in any language you like, to decode the convolutional code discussed in the lectures. The detailed description of the code can be found in the slides.

A dataset is given in the plain text format with index 7 on Canvas within data7.tar.gz. Each set contains 5 files:

* The data file contains 64 lines where each line is the data bit value, either 0 or 1.
* The clean sample file contains 192 lines, each line is the encoded bits, either -1 or 1.
* The samples file contains 192 lines, each line is the encoded bits, originally either -1 or 1, but corrupted by Gaussian noise with zero mean and variance 1.
* The hd file contains the outputs of the decoder using hard decoding.
* The sd file contains the outputs of the decoder using soft decoding.

The last 2 bits of the data are always 0. The datasets that will be used for grading will have the same format.

You should implement the Viterbi algorithm for both hard decoding and soft decoding. **Your program should read the 192 lines in the samples file and output a file in the same format as the data file, i.e., with 64 lines of binary numbers.** If the decoding is successful, the output file should be identical to the data file.

As the noise is intense, you may find that the hard decoder cannot remove all errors but the soft decoder can.

You are free to use the following code as reference:

memlen = 2;

memlenp1 = memlen + 1;

ratio = 3;

regs = zeros(1,memlen);

SNUM = ceil(power(2,memlen));

ENUM = 2\*SNUM;

encoderout = zeros(ENUM,ratio); %encoderout(j,1:3): if binary of j is j0, j1, j2 from the most recent to oldest, the encoder should spit out what 3 bits

for j=0:ENUM-1

thiscombtemp = dec2bin(j,memlenp1); thiscomb = thiscombtemp(memlenp1:-1:1) - '0';

currencodedbits = zeros(1,ratio);

currencodedbits(1) = mod(thiscomb(3) + thiscomb(2) + thiscomb(1), 2);

currencodedbits(2) = mod(thiscomb(3) + thiscomb(1), 2);

currencodedbits(3) = mod(thiscomb(2) + thiscomb(1), 2);

encoderout(j+1,1:ratio) = currencodedbits(1:ratio);

end

if DO\_ENCODING

encodedbits = [];

for j=1:length(databits)

thiscomb = zeros(1,memlenp1); % clear first

for jj=1:memlen

thiscomb(jj+1) = regs(jj);

end

thiscomb(1) = databits(j);

tempp = dec2bin(0,memlenp1); for zzz=1:memlenp1 tempp(zzz) = tempp(zzz) + thiscomb(memlenp1+1-zzz); end

thiscombidx = bin2dec(tempp) + 1;

currencodedbits = encoderout(thiscombidx,1:ratio);

for jj=1:memlen-1

jjj=memlen - jj + 1;

regs(jjj) = regs(jjj-1);

end

regs(1) = databits(j);

encodedbits((j-1)\*ratio+1:j\*ratio) = currencodedbits;

end

end

% decoding part

BNUM = length(databits);

selectedpath = zeros(SNUM,BNUM); % after decoding step j, candpath(2a+b,1:j) is the one of the paths from bit 1 to bit j that arrived at state a

selpathcost = zeros(1,SNUM); selpathcost(2:SNUM) = BNUM\*ratio; %making sure that only paths coming out of 0 can be selected

for j=1:BNUM

thisrcv = zeros(1, ratio); thisrcv(1:ratio) = encodedbits((j-1)\*ratio+1:j\*ratio);

candpaths = zeros(ENUM,BNUM); % after decoding step j, candpath(2a+b,1:j) is the one of the paths from bit 1 to bit j that arrived at state a

candpathcost = zeros(1,ENUM) - 1;

for s=0:SNUM-1

% go from the current state, to two other states

end

for s=0:SNUM-1

% here, go through the candidate paths for each state, select one

end

if 1

fprintf(1,'step %d: [', j)

for zzz=1:SNUM

fprintf(1,'%d', selpathcost(zzz)/2);

if zzz < SNUM

fprintf(1,', ');

end

end

fprintf(1,']\n');

for zzz=1:SNUM

fprintf(1,'S%d:', zzz-1)

for yyy=1:j

fprintf(1,'%d', selectedpath(zzz,yyy+1))

end

fprintf(1,'\n');

end

end

end

decoderout = selectedpath(1,2:BNUM+1);

errnum = sum(abs(decoderout-databits(1:BNUM)))

**Hard Decoding**

step 1 -- samples: [0.317848, 3.018292, 1.454343]; cost: [3.000000, 0.000000, 97.000000, 98.000000]

S0:0

S1:1

S2:0

S3:1

step 2 -- samples: [-1.042104, 1.458167, -1.264144]; cost: [4.000000, 5.000000, 3.000000, 0.000000]

S0:00

S1:01

S2:10

S3:11

step 3 -- samples: [0.708015, -1.386798, -1.553993]; cost: [4.000000, 5.000000, 3.000000, 0.000000]

S0:100

S1:101

S2:110

S3:111

step 4 -- samples: [0.056272, -1.059848, -1.574685]; cost: [4.000000, 5.000000, 3.000000, 0.000000]

S0:1100

S1:1101

S2:1110

S3:1111

step 5 -- samples: [0.441855, 1.235527, 2.021377]; cost: [4.000000, 4.000000, 1.000000, 2.000000]

S0:11100

S1:11001

S2:11110

S3:11111

step 6 -- samples: [0.860169, 0.918449, 0.258247]; cost: [2.000000, 3.000000, 3.000000, 4.000000]

S0:111100

S1:111101

S2:111110

S3:111111

step 7 -- samples: [2.468534, 1.032941, 2.692478]; cost: [4.000000, 2.000000, 4.000000, 5.000000]

S0:1111100

S1:1111001

S2:1111010

S3:1111011

step 8 -- samples: [1.475729, -0.631470, 3.192001]; cost: [6.000000, 5.000000, 2.000000, 5.000000]

S0:11111000

S1:11111001

S2:11110010

S3:11110011

step 9 -- samples: [0.535685, -0.092740, 1.402260]; cost: [4.000000, 3.000000, 5.000000, 6.000000]

S0:111100100

S1:111100101

S2:111110010

S3:111100111

step 10 -- samples: [-1.538382, 0.884785, 0.822000]; cost: [6.000000, 5.000000, 5.000000, 4.000000]

S0:1111001000

S1:1111001001

S2:1111001010

S3:1111001011

step 11 -- samples: [-1.021507, -1.636977, -1.188803]; cost: [6.000000, 6.000000, 6.000000, 5.000000]

S0:11110010000

S1:11110010101

S2:11110010110

S3:11110010111

step 12 -- samples: [-0.851725, -0.783880, 1.529068]; cost: [7.000000, 6.000000, 6.000000, 7.000000]

S0:111100100000

S1:111100101101

S2:111100101110

S3:111100101111

step 13 -- samples: [1.729732, 0.059865, -1.259287]; cost: [6.000000, 8.000000, 8.000000, 7.000000]

S0:1111001011100

S1:1111001000001

S2:1111001011010

S3:1111001011011

step 14 -- samples: [-0.213545, 2.624550, 1.373512]; cost: [8.000000, 7.000000, 7.000000, 9.000000]

S0:11110010111000

S1:11110010111001

S2:11110010110110

S3:11110010000011

step 15 -- samples: [0.749108, -1.925456, 1.098660]; cost: [9.000000, 8.000000, 7.000000, 10.000000]

S0:111100101101100

S1:111100101101101

S2:111100101110010

S3:111100101110011

step 16 -- samples: [0.684280, -0.724741, 1.353289]; cost: [9.000000, 8.000000, 8.000000, 11.000000]

S0:1111001011100100

S1:1111001011100101

S2:1111001011011010

S3:1111001011011011

step 17 -- samples: [-0.200376, -0.833871, 1.776162]; cost: [10.000000, 8.000000, 9.000000, 10.000000]

S0:11110010111001000

S1:11110010110110101

S2:11110010111001010

S3:11110010111001011

step 18 -- samples: [-2.381435, 0.128279, 3.465440]; cost: [11.000000, 10.000000, 10.000000, 9.000000]

S0:111100101110010100

S1:111100101110010101

S2:111100101101101010

S3:111100101101101011

step 19 -- samples: [-0.557917, -3.066648, 0.927401]; cost: [12.000000, 10.000000, 10.000000, 11.000000]

S0:1111001011100101000

S1:1111001011011010101

S2:1111001011011010110

S3:1111001011011010111

step 20 -- samples: [-0.240889, -1.081664, 0.841527]; cost: [13.000000, 10.000000, 11.000000, 12.000000]

S0:11110010111001010000

S1:11110010110110101101

S2:11110010110110101010

S3:11110010110110101011

step 21 -- samples: [-0.993502, -0.064608, -2.643915]; cost: [13.000000, 12.000000, 12.000000, 11.000000]

S0:111100101110010100000

S1:111100101101101010101

S2:111100101101101011010

S3:111100101101101011011

step 22 -- samples: [2.581550, -0.707386, -2.187609]; cost: [13.000000, 14.000000, 13.000000, 11.000000]

S0:1111001011011010110100

S1:1111001011011010110101

S2:1111001011011010101010

S3:1111001011011010110111

step 23 -- samples: [0.075937, -0.297372, 0.878230]; cost: [15.000000, 14.000000, 13.000000, 12.000000]

S0:11110010110110101101000

S1:11110010110110101101001

S2:11110010110110101101110

S3:11110010110110101101111

step 24 -- samples: [1.740772, 0.159282, -1.349471]; cost: [13.000000, 16.000000, 14.000000, 13.000000]

S0:111100101101101011011100

S1:111100101101101011010001

S2:111100101101101011011110

S3:111100101101101011011111

step 25 -- samples: [1.029632, 2.952373, 1.459024]; cost: [15.000000, 13.000000, 14.000000, 15.000000]

S0:1111001011011010110111100

S1:1111001011011010110111001

S2:1111001011011010110111110

S3:1111001011011010110111111

step 26 -- samples: [-1.399312, 0.774615, -1.280283]; cost: [15.000000, 16.000000, 16.000000, 13.000000]

S0:11110010110110101101111100

S1:11110010110110101101111101

S2:11110010110110101101110010

S3:11110010110110101101110011

step 27 -- samples: [0.306725, 0.715625, 0.208290]; cost: [17.000000, 15.000000, 14.000000, 15.000000]

S0:111100101101101011011100100

S1:111100101101101011011111001

S2:111100101101101011011100110

S3:111100101101101011011100111

step 28 -- samples: [-0.759381, -1.094646, 1.793235]; cost: [17.000000, 14.000000, 16.000000, 17.000000]

S0:1111001011011010110111001100

S1:1111001011011010110111001101

S2:1111001011011010110111110010

S3:1111001011011010110111110011

step 29 -- samples: [-0.040560, 2.087103, -0.352831]; cost: [17.000000, 18.000000, 17.000000, 14.000000]

S0:11110010110110101101111100100

S1:11110010110110101101111100101

S2:11110010110110101101110011010

S3:11110010110110101101110011011

step 30 -- samples: [-0.367450, 2.903276, -0.160026]; cost: [18.000000, 19.000000, 15.000000, 16.000000]

S0:111100101101101011011111001000

S1:111100101101101011011111001001

S2:111100101101101011011100110110

S3:111100101101101011011100110111

step 31 -- samples: [-1.347309, -2.695065, 1.919386]; cost: [18.000000, 15.000000, 17.000000, 18.000000]

S0:1111001011011010110111001101100

S1:1111001011011010110111001101101

S2:1111001011011010110111001101110

S3:1111001011011010110111001101111

step 32 -- samples: [1.021531, -1.299340, -0.721016]; cost: [18.000000, 19.000000, 16.000000, 17.000000]

S0:11110010110110101101110011011100

S1:11110010110110101101110011011101

S2:11110010110110101101110011011010

S3:11110010110110101101110011011011

step 33 -- samples: [1.183432, 0.740268, -2.354789]; cost: [16.000000, 19.000000, 19.000000, 18.000000]

S0:111100101101101011011100110110100

S1:111100101101101011011100110111001

S2:111100101101101011011100110110110

S3:111100101101101011011100110110111

step 34 -- samples: [-0.157442, 0.238014, 0.749700]; cost: [18.000000, 17.000000, 18.000000, 20.000000]

S0:1111001011011010110111001101101000

S1:1111001011011010110111001101101001

S2:1111001011011010110111001101101110

S3:1111001011011010110111001101110011

step 35 -- samples: [-2.648763, 0.818009, -0.384313]; cost: [19.000000, 20.000000, 20.000000, 17.000000]

S0:11110010110110101101110011011010000

S1:11110010110110101101110011011010001

S2:11110010110110101101110011011010010

S3:11110010110110101101110011011010011

step 36 -- samples: [-1.376674, 0.301965, 1.304122]; cost: [21.000000, 20.000000, 17.000000, 20.000000]

S0:111100101101101011011100110110100000

S1:111100101101101011011100110110100001

S2:111100101101101011011100110110100110

S3:111100101101101011011100110110100111

step 37 -- samples: [0.101080, -1.446608, 0.533288]; cost: [19.000000, 18.000000, 20.000000, 21.000000]

S0:1111001011011010110111001101101001100

S1:1111001011011010110111001101101001101

S2:1111001011011010110111001101101000010

S3:1111001011011010110111001101101001111

step 38 -- samples: [-0.435821, -1.977716, 1.605895]; cost: [20.000000, 20.000000, 19.000000, 20.000000]

S0:11110010110110101101110011011010011000

S1:11110010110110101101110011011010000101

S2:11110010110110101101110011011010011010

S3:11110010110110101101110011011010011011

step 39 -- samples: [0.886345, 1.764562, -0.330103]; cost: [19.000000, 21.000000, 22.000000, 21.000000]

S0:111100101101101011011100110110100110100

S1:111100101101101011011100110110100110001

S2:111100101101101011011100110110100001010

S3:111100101101101011011100110110100001011

step 40 -- samples: [-0.624216, -0.233347, 0.523472]; cost: [20.000000, 21.000000, 22.000000, 23.000000]

S0:1111001011011010110111001101101001101000

S1:1111001011011010110111001101101001101001

S2:1111001011011010110111001101101001100010

S3:1111001011011010110111001101101001100011

step 41 -- samples: [-0.423176, -0.574027, -1.042931]; cost: [20.000000, 23.000000, 23.000000, 22.000000]

S0:11110010110110101101110011011010011010000

S1:11110010110110101101110011011010011010001

S2:11110010110110101101110011011010011010010

S3:11110010110110101101110011011010011010011

step 42 -- samples: [-2.242263, -1.504952, -0.674149]; cost: [20.000000, 23.000000, 24.000000, 23.000000]

S0:111100101101101011011100110110100110100000

S1:111100101101101011011100110110100110100001

S2:111100101101101011011100110110100110100110

S3:111100101101101011011100110110100110100111

step 43 -- samples: [-2.111423, -0.531816, -0.677387]; cost: [20.000000, 23.000000, 25.000000, 24.000000]

S0:1111001011011010110111001101101001101000000

S1:1111001011011010110111001101101001101000001

S2:1111001011011010110111001101101001101000010

S3:1111001011011010110111001101101001101000011

step 44 -- samples: [-0.899979, -0.698565, -0.976176]; cost: [20.000000, 23.000000, 25.000000, 24.000000]

S0:11110010110110101101110011011010011010000000

S1:11110010110110101101110011011010011010000001

S2:11110010110110101101110011011010011010000010

S3:11110010110110101101110011011010011010000011

step 45 -- samples: [-1.022055, -1.008758, -0.070545]; cost: [20.000000, 23.000000, 25.000000, 24.000000]

S0:111100101101101011011100110110100110100000000

S1:111100101101101011011100110110100110100000001

S2:111100101101101011011100110110100110100000010

S3:111100101101101011011100110110100110100000011

step 46 -- samples: [-1.090432, -3.641146, -1.486150]; cost: [20.000000, 23.000000, 25.000000, 24.000000]

S0:1111001011011010110111001101101001101000000000

S1:1111001011011010110111001101101001101000000001

S2:1111001011011010110111001101101001101000000010

S3:1111001011011010110111001101101001101000000011

step 47 -- samples: [1.195970, 1.959722, 2.380324]; cost: [23.000000, 20.000000, 24.000000, 25.000000]

S0:11110010110110101101110011011010011010000000000

S1:11110010110110101101110011011010011010000000001

S2:11110010110110101101110011011010011010000000010

S3:11110010110110101101110011011010011010000000011

step 48 -- samples: [0.058536, -0.239132, 1.237920]; cost: [25.000000, 24.000000, 20.000000, 23.000000]

S0:111100101101101011011100110110100110100000000000

S1:111100101101101011011100110110100110100000000001

S2:111100101101101011011100110110100110100000000010

S3:111100101101101011011100110110100110100000000011

step 49 -- samples: [-1.208373, -0.975261, 0.971909]; cost: [23.000000, 20.000000, 24.000000, 25.000000]

S0:1111001011011010110111001101101001101000000000100

S1:1111001011011010110111001101101001101000000000101

S2:1111001011011010110111001101101001101000000000110

S3:1111001011011010110111001101101001101000000000111

step 50 -- samples: [1.140359, -0.762414, 0.328451]; cost: [25.000000, 24.000000, 20.000000, 23.000000]

S0:11110010110110101101110011011010011010000000001000

S1:11110010110110101101110011011010011010000000001001

S2:11110010110110101101110011011010011010000000001010

S3:11110010110110101101110011011010011010000000001011

step 51 -- samples: [-0.045000, 1.965767, -1.219757]; cost: [21.000000, 22.000000, 24.000000, 24.000000]

S0:111100101101101011011100110110100110100000000010100

S1:111100101101101011011100110110100110100000000010101

S2:111100101101101011011100110110100110100000000010110

S3:111100101101101011011100110110100110100000000010011

step 52 -- samples: [0.414536, -1.924187, -1.594141]; cost: [22.000000, 23.000000, 23.000000, 24.000000]

S0:1111001011011010110111001101101001101000000000101000

S1:1111001011011010110111001101101001101000000000101001

S2:1111001011011010110111001101101001101000000000101010

S3:1111001011011010110111001101101001101000000000101011

step 53 -- samples: [0.421275, 0.407608, -2.029001]; cost: [23.000000, 23.000000, 25.000000, 24.000000]

S0:11110010110110101101110011011010011010000000001010100

S1:11110010110110101101110011011010011010000000001010001

S2:11110010110110101101110011011010011010000000001010010

S3:11110010110110101101110011011010011010000000001010011

step 54 -- samples: [1.206512, 2.341111, 2.332717]; cost: [26.000000, 23.000000, 24.000000, 25.000000]

S0:111100101101101011011100110110100110100000000010101000

S1:111100101101101011011100110110100110100000000010101001

S2:111100101101101011011100110110100110100000000010100010

S3:111100101101101011011100110110100110100000000010100011

step 55 -- samples: [-0.284895, 0.618376, 1.661644]; cost: [26.000000, 25.000000, 25.000000, 24.000000]

S0:1111001011011010110111001101101001101000000000101000100

S1:1111001011011010110111001101101001101000000000101000101

S2:1111001011011010110111001101101001101000000000101010010

S3:1111001011011010110111001101101001101000000000101010011

step 56 -- samples: [-0.772654, -1.225576, 0.034007]; cost: [27.000000, 25.000000, 25.000000, 26.000000]

S0:11110010110110101101110011011010011010000000001010001000

S1:11110010110110101101110011011010011010000000001010100101

S2:11110010110110101101110011011010011010000000001010100110

S3:11110010110110101101110011011010011010000000001010100111

step 57 -- samples: [-0.904982, 0.743267, 1.310131]; cost: [27.000000, 26.000000, 26.000000, 26.000000]

S0:111100101101101011011100110110100110100000000010101001100

S1:111100101101101011011100110110100110100000000010101001101

S2:111100101101101011011100110110100110100000000010101001110

S3:111100101101101011011100110110100110100000000010101001011

step 58 -- samples: [1.190060, -1.173404, -1.014029]; cost: [27.000000, 28.000000, 27.000000, 26.000000]

S0:1111001011011010110111001101101001101000000000101010011100

S1:1111001011011010110111001101101001101000000000101010011101

S2:1111001011011010110111001101101001101000000000101010011010

S3:1111001011011010110111001101101001101000000000101010010111

step 59 -- samples: [0.387262, 1.071800, -0.362976]; cost: [27.000000, 28.000000, 28.000000, 27.000000]

S0:11110010110110101101110011011010011010000000001010100110100

S1:11110010110110101101110011011010011010000000001010100111001

S2:11110010110110101101110011011010011010000000001010100101110

S3:11110010110110101101110011011010011010000000001010100101111

step 60 -- samples: [1.074894, 0.123267, -1.033128]; cost: [28.000000, 28.000000, 29.000000, 28.000000]

S0:111100101101101011011100110110100110100000000010101001011100

S1:111100101101101011011100110110100110100000000010101001101001

S2:111100101101101011011100110110100110100000000010101001011110

S3:111100101101101011011100110110100110100000000010101001011111

step 61 -- samples: [0.902252, -1.556620, -1.615499]; cost: [29.000000, 30.000000, 29.000000, 28.000000]

S0:1111001011011010110111001101101001101000000000101010010111000

S1:1111001011011010110111001101101001101000000000101010010111001

S2:1111001011011010110111001101101001101000000000101010011010010

S3:1111001011011010110111001101101001101000000000101010010111111

step 62 -- samples: [2.604553, -0.231505, -0.913067]; cost: [30.000000, 31.000000, 31.000000, 28.000000]

S0:11110010110110101101110011011010011010000000001010100101110000

S1:11110010110110101101110011011010011010000000001010100101110001

S2:11110010110110101101110011011010011010000000001010100101110010

S3:11110010110110101101110011011010011010000000001010100101111111

step 63 -- samples: [0.704359, 1.023634, 1.290043]; cost: [32.000000, 30.000000, 29.000000, 30.000000]

S0:111100101101101011011100110110100110100000000010101001011100100

S1:111100101101101011011100110110100110100000000010101001011100001

S2:111100101101101011011100110110100110100000000010101001011111110

S3:111100101101101011011100110110100110100000000010101001011111111

step 64 -- samples: [-0.419899, 1.475274, -2.447319]; cost: [30.000000, 31.000000, 31.000000, 30.000000]

S0:1111001011011010110111001101101001101000000000101010010111111100

S1:1111001011011010110111001101101001101000000000101010010111111101

S2:1111001011011010110111001101101001101000000000101010010111111110

S3:1111001011011010110111001101101001101000000000101010010111000011

errnum = 8

**Soft Decoding**

step 1 -- samples: [0.317848, 3.018292, 1.454343]; cost: [11.953597, 2.372631, 99.008327, 101.917013]

S0:0

S1:1

S2:0

S3:1

step 2 -- samples: [-1.042104, 1.458167, -1.264144]; cost: [15.010662, 16.706824, 10.042192, 2.513362]

S0:00

S1:01

S2:10

S3:11

step 3 -- samples: [0.708015, -1.386798, -1.553993]; cost: [13.086676, 14.837096, 10.081862, 2.784250]

S0:100

S1:101

S2:110

S3:111

step 4 -- samples: [0.056272, -1.059848, -1.574685]; cost: [12.813791, 13.956009, 8.778093, 3.396483]

S0:1100

S1:1101

S2:1110

S3:1111

step 5 -- samples: [0.441855, 1.235527, 2.021377]; cost: [13.525952, 12.837962, 4.985298, 10.615396]

S0:11100

S1:11101

S2:11110

S3:11111

step 6 -- samples: [0.860169, 0.918449, 0.258247]; cost: [5.789993, 8.830735, 12.623935, 13.256989]

S0:111100

S1:111101

S2:111110

S3:111111

step 7 -- samples: [2.468534, 1.032941, 2.692478]; cost: [20.519970, 8.301072, 13.407696, 21.663838]

S0:1111100

S1:1111001

S2:1111010

S3:1111011

step 8 -- samples: [1.475729, -0.631470, 3.192001]; cost: [23.638139, 18.942655, 10.884573, 21.482973]

S0:11110100

S1:11110101

S2:11110010

S3:11110011

step 9 -- samples: [0.535685, -0.092740, 1.402260]; cost: [14.474834, 12.556204, 19.542916, 23.604286]

S0:111100100

S1:111100101

S2:111101010

S3:111101011

step 10 -- samples: [-1.538382, 0.884785, 0.822000]; cost: [18.055810, 17.719004, 17.569944, 14.367610]

S0:1111001000

S1:1111001001

S2:1111001010

S3:1111001011

step 11 -- samples: [-1.021507, -1.636977, -1.188803]; cost: [18.276735, 20.168475, 20.240095, 16.631549]

S0:11110010000

S1:11110010101

S2:11110010110

S3:11110010111

step 12 -- samples: [-0.851725, -0.783880, 1.529068]; cost: [21.509174, 20.414398, 18.373612, 21.567438]

S0:111100100000

S1:111100101101

S2:111100101110

S3:111100101111

step 13 -- samples: [1.729732, 0.059865, -1.259287]; cost: [19.115408, 24.769544, 23.794498, 22.428964]

S0:1111001011100

S1:1111001000001

S2:1111001011010

S3:1111001011111

step 14 -- samples: [-0.213545, 2.624550, 1.373512]; cost: [28.667205, 21.241091, 24.127557, 29.215161]

S0:11110010110100

S1:11110010111001

S2:11110010111110

S3:11110010000011

step 15 -- samples: [0.749108, -1.925456, 1.098660]; cost: [30.640364, 26.090348, 21.705666, 29.252114]

S0:111100101111100

S1:111100101111101

S2:111100101110010

S3:111100101110011

step 16 -- samples: [0.684280, -0.724741, 1.353289]; cost: [26.011855, 23.224355, 26.240477, 31.765097]

S0:1111001011100100

S1:1111001011100101

S2:1111001011111010

S3:1111001011111011

step 17 -- samples: [-0.200376, -0.833871, 1.776162]; cost: [30.198892, 26.875190, 24.259820, 29.079134]

S0:11110010111001000

S1:11110010111110101

S2:11110010111001010

S3:11110010111001011

step 18 -- samples: [-2.381435, 0.128279, 3.465440]; cost: [40.326897, 28.889705, 33.452461, 38.179397]

S0:111100101110010100

S1:111100101110010101

S2:111100101110010110

S3:111100101111101011

step 19 -- samples: [-0.557917, -3.066648, 0.927401]; cost: [44.417570, 35.688332, 32.241410, 39.113674]

S0:1111001011100101000

S1:1111001011100101101

S2:1111001011100101010

S3:1111001011100101011

step 20 -- samples: [-0.240889, -1.081664, 0.841527]; cost: [36.873586, 32.545426, 36.474126, 39.838730]

S0:11110010111001010100

S1:11110010111001010101

S2:11110010111001011010

S3:11110010111001011011

step 21 -- samples: [-0.993502, -0.064608, -2.643915]; cost: [38.662315, 43.550685, 41.608989, 34.463371]

S0:111100101110010101000

S1:111100101110010110101

S2:111100101110010101010

S3:111100101110010101011

step 22 -- samples: [2.581550, -0.707386, -2.187609]; cost: [45.022430, 46.450974, 47.415130, 36.462040]

S0:1111001011100101010100

S1:1111001011100101010001

S2:1111001011100101010110

S3:1111001011100101010111

step 23 -- samples: [0.075937, -0.297372, 0.878230]; cost: [47.611967, 46.298377, 37.889861, 38.899703]

S0:11110010111001010101000

S1:11110010111001010101001

S2:11110010111001010101110

S3:11110010111001010101111

step 24 -- samples: [1.740772, 0.159282, -1.349471]; cost: [38.578701, 45.077751, 45.769029, 39.907107]

S0:111100101110010101011100

S1:111100101110010101011101

S2:111100101110010101011110

S3:111100101110010101011111

step 25 -- samples: [1.029632, 2.952373, 1.459024]; cost: [50.698748, 40.590372, 43.978042, 50.741572]

S0:1111001011100101010111100

S1:1111001011100101010111001

S2:1111001011100101010111110

S3:1111001011100101010111111

step 26 -- samples: [-1.399312, 0.774615, -1.280283]; cost: [46.921069, 48.232241, 47.643195, 40.734775]

S0:11110010111001010101111100

S1:11110010111001010101111101

S2:11110010111001010101110010

S3:11110010111001010101110011

step 27 -- samples: [0.306725, 0.715625, 0.208290]; cost: [48.653927, 47.515221, 41.942377, 43.176757]

S0:111100101110010101011100100

S1:111100101110010101011111001

S2:111100101110010101011100110

S3:111100101110010101011100111

step 28 -- samples: [-0.759381, -1.094646, 1.793235]; cost: [49.584940, 42.290416, 45.714088, 48.630028]

S0:1111001011100101010111001100

S1:1111001011100101010111001101

S2:1111001011100101010111001110

S3:1111001011100101010111001111

step 29 -- samples: [-0.040560, 2.087103, -0.352831]; cost: [47.055781, 51.632295, 48.511977, 43.550989]

S0:11110010111001010101110011100

S1:11110010111001010101110011001

S2:11110010111001010101110011010

S3:11110010111001010101110011011

step 30 -- samples: [-0.367450, 2.903276, -0.160026]; cost: [51.610945, 50.474801, 46.235109, 52.456509]

S0:111100101110010101011100110100

S1:111100101110010101011100111001

S2:111100101110010101011100110110

S3:111100101110010101011100110111

step 31 -- samples: [-1.347309, -2.695065, 1.919386]; cost: [57.369286, 48.154678, 55.088988, 60.909468]

S0:1111001011100101010111001101000

S1:1111001011100101010111001101101

S2:1111001011100101010111001110010

S3:1111001011100101010111001101111

step 32 -- samples: [1.021531, -1.299340, -0.721016]; cost: [57.771618, 58.658032, 49.680660, 52.880370]

S0:11110010111001010101110011100100

S1:11110010111001010101110011100101

S2:11110010111001010101110011011010

S3:11110010111001010101110011011011

step 33 -- samples: [1.183432, 0.740268, -2.354789]; cost: [50.648941, 59.205919, 60.925093, 55.329187]

S0:111100101110010101011100110110100

S1:111100101110010101011100110110101

S2:111100101110010101011100110110110

S3:111100101110010101011100110110111

step 34 -- samples: [-0.157442, 0.238014, 0.749700]; cost: [53.300957, 51.640413, 56.005775, 58.296087]

S0:1111001011100101010111001101101000

S1:1111001011100101010111001101101001

S2:1111001011100101010111001101101110

S3:1111001011100101010111001101101111

step 35 -- samples: [-2.648763, 0.818009, -0.384313]; cost: [56.502281, 59.975725, 60.630019, 53.205719]

S0:11110010111001010101110011011010000

S1:11110010111001010101110011011011101

S2:11110010111001010101110011011011110

S3:11110010111001010101110011011010011

step 36 -- samples: [-1.376674, 0.301965, 1.304122]; cost: [60.075268, 59.616442, 53.566532, 59.532054]

S0:111100101110010101011100110110100000

S1:111100101110010101011100110110100001

S2:111100101110010101011100110110100110

S3:111100101110010101011100110110100111

step 37 -- samples: [0.101080, -1.446608, 0.533288]; cost: [58.138992, 54.381360, 60.229110, 61.211298]

S0:1111001011100101010111001101101001100

S1:1111001011100101010111001101101001101

S2:1111001011100101010111001101101000010

S3:1111001011100101010111001101101001111

step 38 -- samples: [-0.435821, -1.977716, 1.605895]; cost: [62.171449, 61.049777, 56.073669, 62.369249]

S0:11110010111001010101110011011010011000

S1:11110010111001010101110011011010000101

S2:11110010111001010101110011011010011010

S3:11110010111001010101110011011010011011

step 39 -- samples: [0.886345, 1.764562, -0.330103]; cost: [56.596787, 62.558807, 65.325263, 63.345585]

S0:111100101110010101011100110110100110100

S1:111100101110010101011100110110100110101

S2:111100101110010101011100110110100110110

S3:111100101110010101011100110110100001011

step 40 -- samples: [-0.624216, -0.233347, 0.523472]; cost: [58.121755, 58.789937, 64.285263, 64.550469]

S0:1111001011100101010111001101101001101000

S1:1111001011100101010111001101101001101001

S2:1111001011100101010111001101101001101010

S3:1111001011100101010111001101101001101011

step 41 -- samples: [-0.423176, -0.574027, -1.042931]; cost: [58.379766, 62.460034, 61.980162, 60.196002]

S0:11110010111001010101110011011010011010000

S1:11110010111001010101110011011010011010001

S2:11110010111001010101110011011010011010010

S3:11110010111001010101110011011010011010011

step 42 -- samples: [-2.242263, -1.504952, -0.674149]; cost: [59.331953, 64.280647, 65.506391, 65.632715]

S0:111100101110010101011100110110100110100000

S1:111100101110010101011100110110100110100101

S2:111100101110010101011100110110100110100110

S3:111100101110010101011100110110100110100111

step 43 -- samples: [-2.111423, -0.531816, -0.677387]; cost: [60.111221, 66.752473, 68.830389, 66.123547]

S0:1111001011100101010111001101101001101000000

S1:1111001011100101010111001101101001101000001

S2:1111001011100101010111001101101001101001110

S3:1111001011100101010111001101101001101001011

step 44 -- samples: [-0.899979, -0.698565, -0.976176]; cost: [60.161938, 65.311378, 69.523746, 67.974222]

S0:11110010111001010101110011011010011010000000

S1:11110010111001010101110011011010011010000001

S2:11110010111001010101110011011010011010010110

S3:11110010111001010101110011011010011010010111

step 45 -- samples: [-1.022055, -1.008758, -0.070545]; cost: [60.594163, 64.796879, 67.928803, 67.761119]

S0:111100101110010101011100110110100110100000000

S1:111100101110010101011100110110100110100000001

S2:111100101110010101011100110110100110100000010

S3:111100101110010101011100110110100110100000011

step 46 -- samples: [-1.090432, -3.641146, -1.486150]; cost: [64.204249, 74.511189, 73.560129, 73.552069]

S0:1111001011100101010111001101101001101000000000

S1:1111001011100101010111001101101001101000000101

S2:1111001011100101010111001101101001101000000010

S3:1111001011100101010111001101101001101000000111

step 47 -- samples: [1.195970, 1.959722, 2.380324]; cost: [76.708664, 65.636632, 77.376392, 83.096160]

S0:11110010111001010101110011011010011010000000000

S1:11110010111001010101110011011010011010000000001

S2:11110010111001010101110011011010011010000001110

S3:11110010111001010101110011011010011010000001011

step 48 -- samples: [0.058536, -0.239132, 1.237920]; cost: [80.062516, 77.947868, 66.397572, 69.468748]

S0:111100101110010101011100110110100110100000000000

S1:111100101110010101011100110110100110100000000001

S2:111100101110010101011100110110100110100000000010

S3:111100101110010101011100110110100110100000000011

step 49 -- samples: [-1.208373, -0.975261, 0.971909]; cost: [72.731068, 66.419982, 71.441680, 73.851722]

S0:1111001011100101010111001101101001101000000000100

S1:1111001011100101010111001101101001101000000000101

S2:1111001011100101010111001101101001101000000000110

S3:1111001011100101010111001101101001101000000000111

step 50 -- samples: [1.140359, -0.762414, 0.328451]; cost: [73.886973, 73.985961, 66.683545, 71.145993]

S0:11110010111001010101110011011010011010000000001100

S1:11110010111001010101110011011010011010000000001101

S2:11110010111001010101110011011010011010000000001010

S3:11110010111001010101110011011010011010000000001011

step 51 -- samples: [-0.045000, 1.965767, -1.219757]; cost: [67.720057, 74.001105, 74.532019, 74.932473]

S0:111100101110010101011100110110100110100000000010100

S1:111100101110010101011100110110100110100000000010101

S2:111100101110010101011100110110100110100000000010110

S3:111100101110010101011100110110100110100000000011011

step 52 -- samples: [0.414536, -1.924187, -1.594141]; cost: [69.324076, 75.531660, 77.964334, 75.707420]

S0:1111001011100101010111001101101001101000000000101000

S1:1111001011100101010111001101101001101000000000101001

S2:1111001011100101010111001101101001101000000000101010

S3:1111001011100101010111001101101001101000000000110111

step 53 -- samples: [0.421275, 0.407608, -2.029001]; cost: [71.854189, 74.254425, 81.277225, 77.246557]

S0:11110010111001010101110011011010011010000000001010000

S1:11110010111001010101110011011010011010000000001010001

S2:11110010111001010101110011011010011010000000001010010

S3:11110010111001010101110011011010011010000000001010011

step 54 -- samples: [1.206512, 2.341111, 2.332717]; cost: [85.423549, 73.662869, 80.745327, 83.141563]

S0:111100101110010101011100110110100110100000000010100000

S1:111100101110010101011100110110100110100000000010100001

S2:111100101110010101011100110110100110100000000010100010

S3:111100101110010101011100110110100110100000000010100011

step 55 -- samples: [-0.284895, 0.618376, 1.661644]; cost: [85.185797, 82.529471, 76.016803, 77.533549]

S0:1111001011100101010111001101101001101000000000101000100

S1:1111001011100101010111001101101001101000000000101000101

S2:1111001011100101010111001101101001101000000000101000010

S3:1111001011100101010111001101101001101000000000101000011

step 56 -- samples: [-0.772654, -1.225576, 0.034007]; cost: [80.599134, 76.534660, 80.502558, 79.664728]

S0:11110010111001010101110011011010011010000000001010000100

S1:11110010111001010101110011011010011010000000001010000101

S2:11110010111001010101110011011010011010000000001010000110

S3:11110010111001010101110011011010011010000000001010000111

step 57 -- samples: [-0.904982, 0.743267, 1.310131]; cost: [84.791491, 82.074653, 79.750289, 79.240483]

S0:111100101110010101011100110110100110100000000010100001000

S1:111100101110010101011100110110100110100000000010100001101

S2:111100101110010101011100110110100110100000000010100001110

S3:111100101110010101011100110110100110100000000010100001011

step 58 -- samples: [1.190060, -1.173404, -1.014029]; cost: [82.130291, 84.191661, 84.135905, 79.273677]

S0:1111001011100101010111001101101001101000000000101000011100

S1:1111001011100101010111001101101001101000000000101000011101

S2:1111001011100101010111001101101001101000000000101000011010

S3:1111001011100101010111001101101001101000000000101000010111

step 59 -- samples: [0.387262, 1.071800, -0.362976]; cost: [84.529106, 83.249444, 81.167354, 81.810478]

S0:11110010111001010101110011011010011010000000001010000110100

S1:11110010111001010101110011011010011010000000001010000111001

S2:11110010111001010101110011011010011010000000001010000101110

S3:11110010111001010101110011011010011010000000001010000101111

step 60 -- samples: [1.074894, 0.123267, -1.033128]; cost: [81.555038, 86.017616, 85.949918, 82.444696]

S0:111100101110010101011100110110100110100000000010100001011100

S1:111100101110010101011100110110100110100000000010100001011101

S2:111100101110010101011100110110100110100000000010100001110010

S3:111100101110010101011100110110100110100000000010100001011111

step 61 -- samples: [0.902252, -1.556620, -1.615499]; cost: [83.708652, 88.248386, 89.597724, 82.793806]

S0:1111001011100101010111001101101001101000000000101000010111000

S1:1111001011100101010111001101101001101000000000101000010111001

S2:1111001011100101010111001101101001101000000000101000010111010

S3:1111001011100101010111001101101001101000000000101000010111111

step 62 -- samples: [2.604553, -0.231505, -0.913067]; cost: [90.504124, 87.584162, 91.660886, 84.380172]

S0:11110010111001010101110011011010011010000000001010000101110000

S1:11110010111001010101110011011010011010000000001010000101110001

S2:11110010111001010101110011011010011010000000001010000101110010

S3:11110010111001010101110011011010011010000000001010000101111111

step 63 -- samples: [0.704359, 1.023634, 1.290043]; cost: [94.327015, 90.590167, 85.874933, 89.093569]

S0:111100101110010101011100110110100110100000000010100001011100100

S1:111100101110010101011100110110100110100000000010100001011100001

S2:111100101110010101011100110110100110100000000010100001011111110

S3:111100101110010101011100110110100110100000000010100001011111111

step 64 -- samples: [-0.419899, 1.475274, -2.447319]; cost: [88.043299, 95.048687, 95.316775, 91.918735]

S0:1111001011100101010111001101101001101000000000101000010111111100

S1:1111001011100101010111001101101001101000000000101000010111111101

S2:1111001011100101010111001101101001101000000000101000010111111110

S3:1111001011100101010111001101101001101000000000101000010111000011

errnum = 0